C | EN

(1) a) $\overrightarrow{AB} = \begin{pmatrix} -2 \\ 6 \\ -5 \end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$

$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} \sim \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ $\qquad \mathcal{E}_1 : x + 2y + 2z = D$ 1

$B \in \mathcal{E}_1 : 4 - 4 = \underline{\underline{D = 0}}$ 1

$\vec{n}_1 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ $\vec{n}_2 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ 1

$|\cdot\cdot| = 3$ $\qquad |\cdots| = \sqrt{6}$ $\qquad \cos\varphi = \dfrac{1-2+4}{3\sqrt{6}} \Rightarrow \varphi \approx 65.91°$

b) $\vec{r}_{M_{1/2}} = \pm 3 \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ 1 $\quad M_1(3/6/6) \quad M_2(-3/-6/-6)$

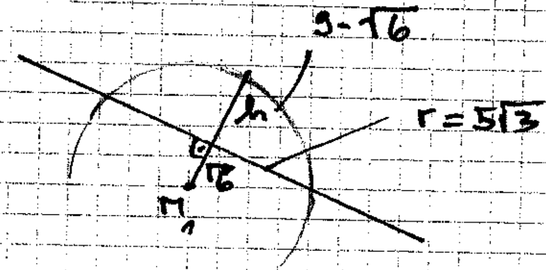
$K_{1,2} : (x \mp 3)^2 + (y \mp 6)^2 + (z \mp 6)^2 = 9^2$ 1

c) Abstand $M_{1/2}$ zu $\mathcal{E}_2$ mit Lot, HNF, Spatprod.

zB. $\ell : \vec{r} = \begin{pmatrix} 3 \\ 6 \\ 6 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} \cap \mathcal{E}_2$ 2

$(3+t) - (6-t) + 2(6+2t) = 15 \Rightarrow t = 1$ Abst. $\sqrt{6}$ 1

$\Rightarrow \mathcal{E}_2$ schneidet K_1, $r_{\text{Schnittkreis}} = \sqrt{81-6} = 5\sqrt{3}$ 1



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$V_\triangle \overset{FS}{=} \frac{1}{3}\pi h^2 (3R - h) \approx 918.89$ resp. 30.1 % des Kugelvolumens 1

Verhältnis: $\qquad V_\triangle / V_\square = \dfrac{30.1}{69.9} \approx 0.43$

Abstand M_2 zu $\mathcal{E}_2$: $\left| \dfrac{-3+6-12-15}{\sqrt{6}} \right| = 4\sqrt{6} > 9$

$\mathcal{E}_2$ schneidet K_2 nicht! 1

(2) a) $y(-x) = -y(x) \Rightarrow$ Symm. zum 0-Pkt. 1

[C|E]

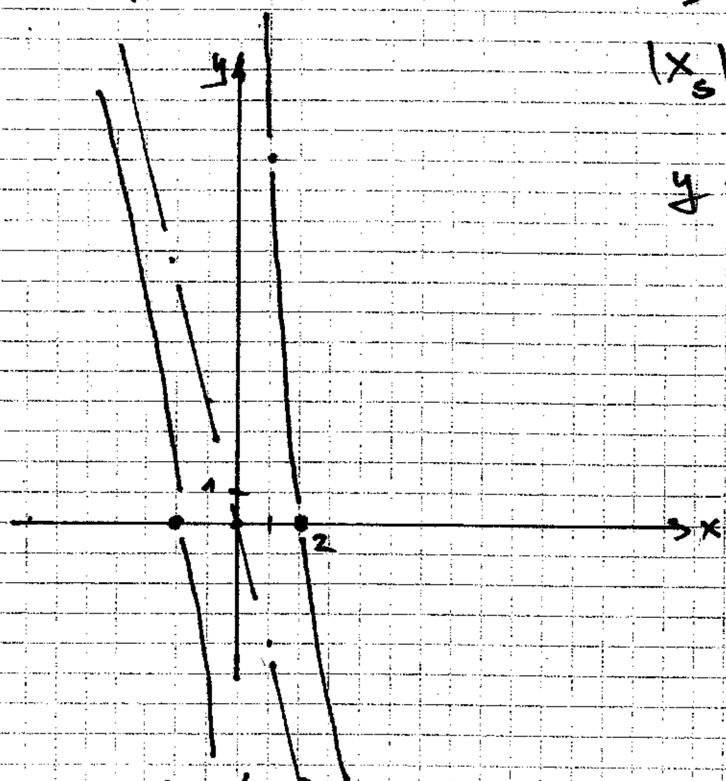
Vertikale Asymptote $x = 0$

schiefe " $y = -4x$ 1

$y = 0 \Rightarrow a^2 = 4x^2$ $x_s = \pm \frac{a}{2}$

$|x_s| = 2 \Rightarrow \underline{a = 4}$ 1

$y = \frac{16}{x} - 4x$



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b) $P(u/v) \in K$

$u^2 + v^2$ soll minimal werden! 1

$f(u) := u^2 + \left(\frac{16}{u} - 4u\right)^2$ 1 + 1

$\frac{df}{du} = 2u + 2\left(\frac{16}{u} - 4u\right)\left(-\frac{16}{u^2} - 4\right) = 0\,!$ 2

$\Rightarrow u - \frac{16^2}{u^3} - \cancel{\frac{64}{u}} + \cancel{\frac{64}{u}} + 16u = 0$

$\Rightarrow 17u = \frac{16^2}{u^3}$ $u^4 = \frac{16^2}{17}$ $u = \pm\frac{4}{\sqrt[4]{17}}$ 1

$\Rightarrow P(\pm 1.970 / \pm 0.243)$ 1

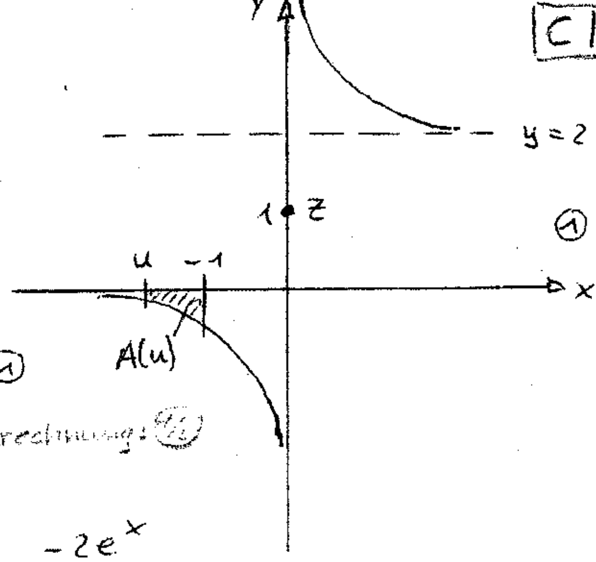
Minimum geom. klar, resp. $\frac{d^2f}{du^2} > 0$ 1

geom. Lös.: $y'_{(P)} \cdot m_{OP} = -1$

$\Rightarrow \left(-\frac{16}{x^2} - 4\right)\left(\frac{y}{x}\right) = -1$

$\Rightarrow \left(-\frac{16}{x^2} - 4\right)\left(\frac{16}{x^2} - 4\right) = -1$ .... $x^4 = \frac{16^2}{17}$

③ a) $\mathbb{D} = \mathbb{R} \setminus \{0\}$ (1/2)



|C|EM|

Asymptoten: $x = 0$

$y = 2$ ①

$y = 0$

(1)

b) $Z(0|1)$ (1/2)

$$\frac{2e^{x}}{e^{x}-1} - 1 = -\left(\frac{2e^{-x}}{e^{-x}-1} - 1\right)$$ ①

Ausrechnung: (1/2)

c) $$f'(x) = \frac{2e^{x}(e^{x}-1) - 2e^{x}\cdot e^{x}}{(e^{x}-1)^2} = \frac{-2e^{x}}{(e^{x}-1)^2}$$ ①

Tangente: $$y = -\frac{2e}{(e-1)^2}x + \frac{2e}{(e-1)^2} + \frac{2e}{e-1} = -\frac{2e}{(e-1)^2}x + \frac{2e^2}{(e-1)^2}$$ ①

Schnittpunkt mit der x-Achse: $S(e/0)$ (1/2)

$$\tan\alpha = \left|-\frac{2e}{(e-1)^2}\right| \qquad \alpha \approx 61{,}49°$$ ①

d) $$A(u) = -\int_u^{-1} \frac{2e^{x}}{e^{x}-1}\,dx = -\Big[2\ln|e^{x}-1|\Big]_u^{-1} = -2\ln\left(1-\frac{1}{e}\right) + 2\ln(1-e^{u})$$ ① ②

$$\int \frac{2e^{x}}{e^{x}-1}\,dx = \int \frac{2}{u}\,du = 2\ln|u| + C$$

$$\lim_{u \to -\infty} A(u) = -2\ln\left(1-\frac{1}{e}\right) = 2\ln\left(\frac{e}{e-1}\right)$$ ①

$u = e^{x} - 1$

$\frac{du}{dx} = e^{x} \qquad dx = \frac{du}{e^{x}}$

(4) 1 a) $$z = 2\bar{z}\cdot(i - z)$$ CIEN

$$x + iy = 2(x - iy)(i - x - iy)$$
$$= 2\left[ix - x^2 - \cancel{ixy} + y + \cancel{ixy} - y^2\right]$$

$$\Rightarrow \quad x = 2(y - x^2 - y^2)$$ 1

$$y = 2x$$ 1

$$\Rightarrow \quad x = 2(2x - x^2 - 4x^2) = 4x - 10x^2$$

$$\Rightarrow \quad 0 = 3x - 10x^2 \quad \begin{cases} x_1 = 0 & y_1 = 0 \\ x_2 = \frac{3}{10} & y_2 = \frac{6}{10} \end{cases}$$ 1/2 1

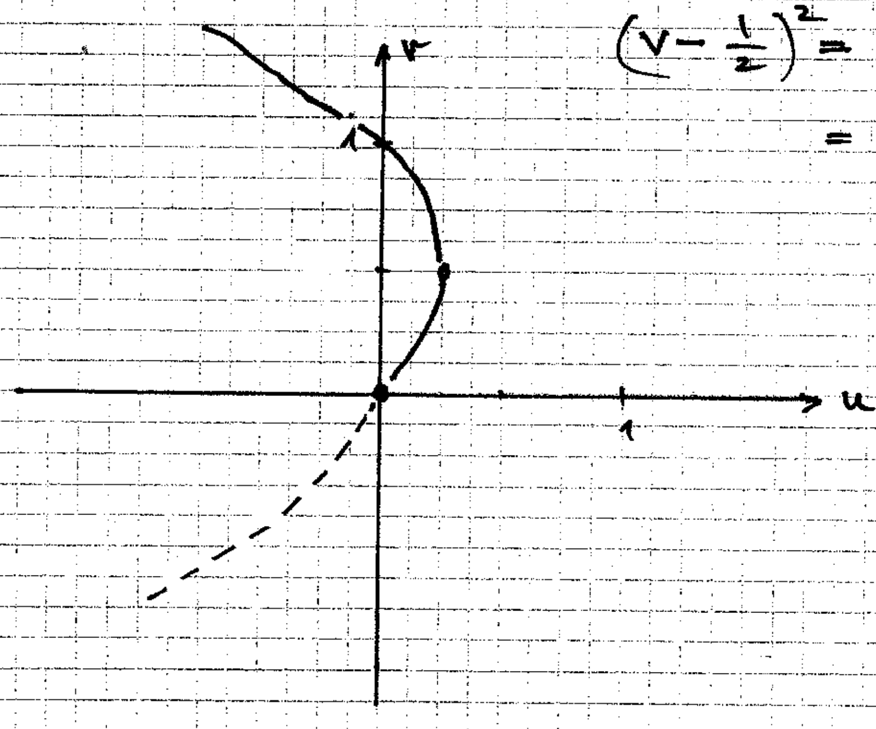
b) $$g: \; y = x \; , \; x, y \geq 0$$

$$u + iv = 2(x - ix)(i - x - ix)$$
$$= 2\left[ix - 2x^2 + x\right]$$

d.h. $$\begin{cases} u = 2(x - 2x^2) = 2x - 4x^2 \\ v = 2x \end{cases}$$ 1

$$\Rightarrow \quad u = v - v^2 \qquad v^2 - v = -u$$

$$\left(v - \frac{1}{2}\right)^2 = -u + \frac{1}{4}$$ 1
$$= -\left(u - \frac{1}{4}\right)$$



1/2

④ 2. a) $\det(A - \lambda I) = 0$

CIEN

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 0 \\ 0 & 0 & -1-\lambda \end{vmatrix} = 0$$

$$-\lambda^2(1+\lambda) + (1+\lambda) = (1+\lambda)(1-\lambda^2) = 0 \qquad \lambda_{1/2} = 1 \mid -1$$

$$\lambda_1 = 1: \begin{pmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \qquad \begin{matrix} -x+y=0 \\ x-y=0 \\ -2z=0 \end{matrix} \qquad \vec{V_1} = t\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\lambda_2 = -1: \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 \qquad \begin{matrix} x+y=0 \\ x+y=0 \\ 0z=0 \end{matrix} \qquad \vec{V_2} = t\begin{pmatrix} 1 \\ -1 \\ u \end{pmatrix}$$

④

b) $\vec{V_1}$ liegen in den Ebenen $z=0$ und $x=y$. Werden auf sich selbst abgebildet.

" $\vec{V_2}$ liegen in der Ebene $x=-y$. Werden am Nullpunkt gespiegelt.

Abbildung: $u = y$, $v = x$, $w = -z$

Spiegelung an den Ebenen $z=0$ und $x=y$

②

⑤ a) 6 Kinder, je 3 Schichten : $3^6 = 729$

b) $\binom{6}{2}\binom{4}{2}\binom{2}{2} = 15 \cdot 6 = 90$ $\quad 2 + 1/2$

c) mindestens 1 Schicht ohne Geburt:

Typ					
0	5	1	$6 \cdot \binom{6}{5}\binom{1}{1}$	=	36
0	4	2	$6 \cdot \binom{6}{4}\binom{2}{2}$	=	90
0	3	3	$3 \cdot \binom{6}{3}\binom{3}{3}$	=	60
0	0	6	3	=	3

1 $\quad$ 1 $\quad$ 1 $\quad$ 1/2

189 $\quad$ 1

$$W(\text{mindesten } 1\times \text{leer}) = \frac{189}{729} \cong 25.93\,\%$$

d) Binomialverteilung mit $p_M = \frac{1}{3}$ $\quad q = \frac{2}{3}$

$n = 12$

1

$$P(\text{Anzahl Mittagsgeb.} > 6)$$

$$= \binom{12}{7}\left(\frac{1}{3}\right)^7\left(\frac{2}{3}\right)^5 + \binom{12}{8}\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^4$$

$$+ \dots + \binom{12}{12}\left(\frac{1}{3}\right)^{12}$$

$$= 1 - F\left(12, \frac{1}{3}, 6\right)$$ $\quad 3$

$$\cong 1 - 0.9336 = 6.645\,\%$$