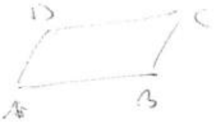
Math EN
#18

1. a) $\vec{AB} = \begin{pmatrix}-1\\-5\\-4\end{pmatrix}$ $\quad \vec{AD} = \begin{pmatrix}2\\-2\\2\end{pmatrix}$ $\quad \vec{AB}\cdot\vec{AD} = 0$ ✓



b) $\vec{r}_C = \vec{r}_B + \vec{AD} = \begin{pmatrix}1\\-5\\-4\end{pmatrix}$ $\quad \underline{C(1|-5|-4)}$

c) $V = 56 = \frac{1}{3}Gh$ $\quad G = |\vec{AB}||\vec{AD}| = \sqrt{42}\cdot\sqrt{12} = 6\sqrt{14}$

$h = 2\sqrt{14}$ $\quad \vec{n} = \vec{AB}\times\vec{AD} = \begin{pmatrix}-18\\-6\\12\end{pmatrix}$ $\quad |\vec{n}| = 6\sqrt{14}$

$\vec{MS} = \pm\frac{1}{3}\vec{n} = \pm\begin{pmatrix}-6\\-2\\4\end{pmatrix}$

$\vec{r}_M = \frac{1}{2}(\vec{r}_A + \vec{r}_C) = \begin{pmatrix}1/2\\-3/2\\-3\end{pmatrix}$ $\quad \vec{r}_{S_{1/2}} = \begin{pmatrix}1/2\\-3/2\\-3\end{pmatrix} \pm \begin{pmatrix}6\\2\\-4\end{pmatrix}$

$S_1(-\frac{11}{2}|-\frac{7}{2}|1)$

$S_2(\frac{13}{2}|\frac{1}{2}|-7)$

d) $E(A,B,S_1)$: $\quad \vec{AS_1} = \begin{pmatrix}-11/2\\-11/2\\3\end{pmatrix}$

$\vec{AS}\times\vec{AB} = \begin{pmatrix}37\\-25\\22\end{pmatrix} = \vec{n}_2$ $\quad E: 37x - 25y + 22z + d = 0$

A: $\quad d = 94$

$E: 37x - 25y + 22z + 94 = 0$

e) $\cos\alpha^* = \frac{\vec{n}\cdot\vec{n}_2}{n\cdot n_2} = \frac{-252}{6\sqrt{14}\sqrt{2478}} \Rightarrow \alpha^* = 103{,}03° \rightarrow \underline{77° = \alpha}$

2. a) $f'(x) = 8x\ln x + 4x$

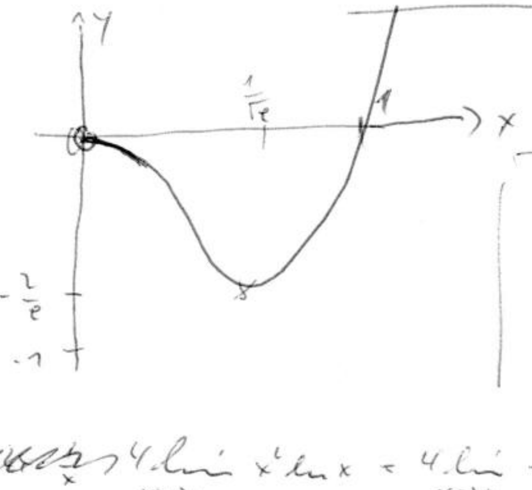
$f''(x) = 8\ln x + 12$

$D = \mathbb{R}^+$, NST: $\underline{x = 1}$

Ext. $\underbrace{4x}_{\substack{(x=0)\\ \notin D}}\underbrace{(2\ln x + 1)} = 0$ $\quad \underline{x = e^{-\frac{1}{2}}}$ $\quad f''(e^{-\frac{1}{2}}) > 0 \Rightarrow \underline{\text{Min}(\frac{1}{\sqrt{e}}|-\frac{2}{e})}$

WP $\quad \underline{x = e^{-\frac{3}{2}}}$, VZW da ln monoton $\rightarrow$ WP$(e^{-3/2}|-\frac{6}{e^3})$

$W = [-\frac{6}{e^3}; \infty[$



Graph geht flach gegen $y=0$ f. $x\to 0$ $\sim x^2$

Vor: $\ln x$, x^2, x^{-1} stetig und diff.bar in $\mathbb{R}^+_0$, $\mathbb{R}^+$

b) $\lim_{x\to 0} f(x) = 4\lim_{x\to 0} x^2\ln x = 4\lim_{x\to 0}\frac{\ln x}{x^{-2}} \overset{l'H}{=} 4\lim\frac{1/x}{-2x^{-3}}$

$= -2\lim_{x\to 0} x^2 = 0$

$\lim_{x\to 0} f'(x) = 4\lim_{x\to 0}(2x\ln x + x) = 8\lim_{x\to 0}\frac{\ln x}{x^{-1}} \overset{l'H}{=} 8\lim_{x\to 0}\frac{1/x}{-x^{-2}} = -8\lim_{x\to 0} x = \underline{0}$

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2c) $A = \int_1^e f(x)\,dx = 4\int_1^e x^2 \ln x\,dx = 4\left(\left[\frac{x^3}{3}\ln x\right]_1^e - \int_1^e \frac{x^3}{3}\cdot\frac{1}{x}\,dx \right)$

$= 4\left(\frac{e^3}{3} - 0 - \left[\frac{1}{9}x^3\right]_1^e\right) = \frac{4}{3}e^3 - \frac{4}{9}e^3 + \frac{4}{9} = \underline{\frac{8}{9}e^3 + \frac{4}{9}}$

d) $\frac{4x^2\ln x - 0}{x - 0} = 8x\ln x + 4x$

$4x^2\ln x + 4x^2 = 0$

$\underbrace{4x^2}_{(x=0)\ \notin D}\underbrace{(\ln x + 1)}_{} = 0$

$x = e^{-1}$ $\quad P\left(\frac{1}{e}\middle|-\frac{4}{e^2}\right) \quad t: \underline{y = -\frac{4}{e}x}$

$y = -\frac{4}{e^2}$

$y' = -\frac{4}{e}$

3. $16r \mid 24b \qquad \frac{2}{5} \mid \frac{3}{5}$

$\qquad\qquad\qquad 0{,}4 \quad 0{,}6$

a) $P(3\text{ gleiche}) = \left(\frac{16}{40}\right)^3 + \left(\frac{24}{40}\right)^3 = \left(\frac{2}{5}\right)^3 + \left(\frac{3}{5}\right)^3 = \underline{\frac{7}{25} = 28\%}$

b) $P(5\text{ mal, mind. 1 rot}) = 1 - P(\text{kein rot}) = 1 - \left(\frac{3}{5}\right)^5 = \underline{92{,}2\%}$

c) $P(\text{mind. ein blau in } n \text{ Zügen}) > 95\%$

$1 - P(\text{kein blau}) > 0{,}95$

$1 - \left(\frac{2}{5}\right)^n > 0{,}95$

$n > \log_{0,4} 0{,}05 = 3{,}3 \qquad \underline{\text{ab 4 Zügen}}$

d) $\underline{E} = 0{,}4\cdot(-5) + 0{,}6\cdot 5 + 0{,}4^2\cdot(-5) + 0{,}4\cdot 0{,}6\cdot 10 + 0{,}4^3\cdot(-5) + 0{,}4^2\cdot 0{,}6\cdot(15)$

$\underline{= 3{,}72}$

e) $\left.\begin{array}{l} P(S) = 0{,}6 + 0{,}4\cdot 0{,}6 + 0{,}4^2\cdot 0{,}6 = 0{,}936 \\ P(S,2) = 0{,}4\cdot 0{,}6 = 0{,}24 \end{array}\right\} \underline{25{,}6\%}$

f) $P(8|9|10 \text{ von } 20 \text{ rot}) = \binom{20}{8}0{,}4^8\,0{,}6^{12} + \binom{20}{9}0{,}4^9\,0{,}6^{11} + \binom{20}{10}0{,}4^{10}\,0{,}6^{10}$

$\underline{\approx 45{,}7\%}$

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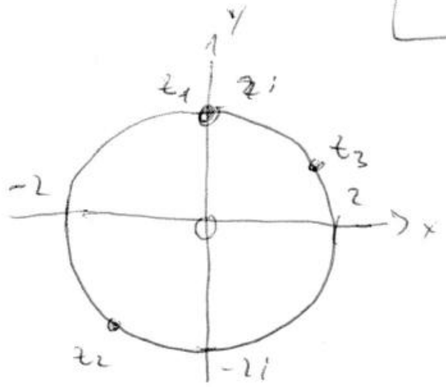
4. I $z_n = 2i\left(-\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^{n-1}$

a) $z_1 = 2i = 2\,\text{cis}(\frac{\pi}{2})$ $\quad +\frac{5}{6}\pi$

$z_2 = -1-\sqrt{3}i = 2\cdot\text{cis}(4\frac{\pi}{3})$ $\quad +\frac{5}{6}\pi$

$z_3 = \sqrt{3}+i = 2\,\text{cis}(\frac{\pi}{6})$



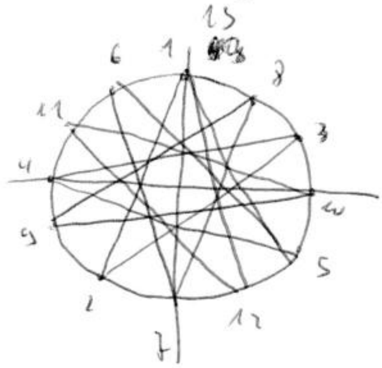
b) Ausgehend von $\frac{\pi}{2}$ (90°) wird mit jedem n um $\frac{5}{6}\pi$ (150°) weiter gedreht. $-\frac{\sqrt{3}}{2}+\frac{1}{2}i = \underline{1}\cdot\text{cis}\left(\frac{5\pi}{6}\right)$ Multiplikation $\to +\frac{5\pi}{6}$

keine Änderung im Betrag

$\frac{5}{6}\pi\cdot n = 2\pi\cdot k$

$5n = 12k$

kgV(5;12) = 60 $\longrightarrow \underline{n = 13}$



c) $12\cdot\sqrt{(1+\sqrt{3})^2+1^2}$

$= \underline{12\cdot\sqrt{5+2\sqrt{3}}}$

II $\quad A = \begin{pmatrix}2&1\\1&2\end{pmatrix}$

a) $A\cdot\binom{1}{0} = \binom{2}{1}$ ✓ $\quad A\binom{0}{1} = \binom{1}{2}$ ✓

b) $A\vec{e} = \lambda\vec{e}$ $\quad \begin{pmatrix}2-\lambda&1\\1&2-\lambda\end{pmatrix}\vec{e} = 0 \quad |\ \det$

$(2-\lambda)^2 = 1$

$\underline{\lambda_1 = 1}$ $\longrightarrow$ $\begin{pmatrix}1&1\\1&1\end{pmatrix}\binom{x}{y} = 0 \to x+y=0 \Rightarrow x=-y \quad \vec{e}_1 = k\binom{1}{-1}$

$\underline{\lambda_2 = 3}$ $\quad \begin{pmatrix}-1&1\\1&-1\end{pmatrix}\binom{x}{y} = 0 \to x-y=0 \Rightarrow x=y \quad \vec{e}_2 = k\binom{1}{1}$

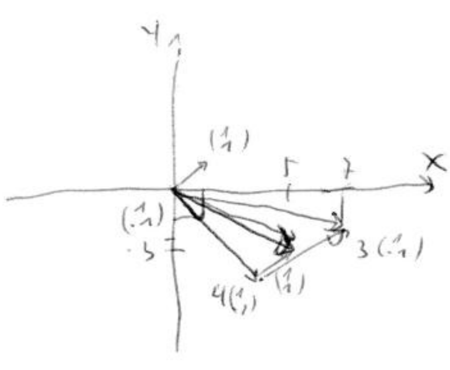
c) g, h sind Eigengeraden

$\begin{pmatrix}2&1\\1&2\end{pmatrix}\binom{x}{x} = \binom{3x}{3x} = 3\binom{x}{x}$ $\qquad \begin{pmatrix}2&1\\1&2\end{pmatrix}\binom{x}{-x} = \binom{x}{-x} = 1\cdot\binom{x}{-x}$

d) $\underset{\vec{a}}{\binom{5}{-3}} = x\underset{\vec{e}_1}{\binom{1}{1}} + y\underset{\vec{e}_2}{\binom{1}{-1}} \longrightarrow \begin{matrix}x=1\\y=4\end{matrix}$

$A\vec{a} = A(\vec{e}_1+4\vec{e}_2) = A\vec{e}_1 + A\vec{e}_2 = 3\vec{e}_1 + 4A\vec{e}_2 = 3\vec{e}_1 + 4\vec{e}_2 = \binom{3}{3}+\binom{4}{-4} = \underline{\binom{7}{-1}}$

$\begin{pmatrix}2&1\\1&2\end{pmatrix}\binom{5}{-3} = \binom{10-3}{5-6} = \binom{7}{-1}$



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5 a) $2\sin\left(2x - \frac{\pi}{2}\right) = 1$

$2x_1 - \frac{\pi}{2} = \frac{\pi}{6} + z\cdot 2\pi$ $\qquad$ $2x_2 - \frac{\pi}{2} = \frac{5}{6}\pi + z\cdot 2\pi$

$x_1 = \frac{\pi}{3} + z\cdot \cancel{2}\pi$ $\qquad$ $x_2 = \frac{2}{3}\pi + z\cdot \cancel{2}\pi$

$\mathbb{L} = \left\{ \frac{\pi}{3}; \frac{4}{3}\pi; \frac{2}{3}\pi; \frac{5}{3}\pi \right\}$

b) $9x^2 - 4y^2 - 36x - 24y - 36 = 0$

$(3x \mp 6)^2 - (2y+6)^2 = 36$

$9(x\mp 2)^2 - 4(y+3)^2 = 36$

$\frac{(x\mp 2)^2}{4} - \frac{(y+3)^2}{9} = 1$

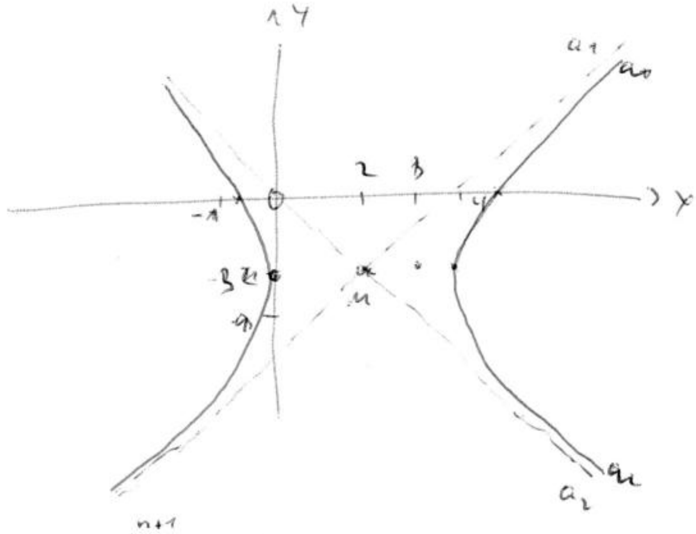
Hyperbel $a = 2$, $b = 3$ um $\binom{2}{-3}$ aus $\binom{0}{0}$ verschoben $M(2|-3)$

$c = \sqrt{a^2+b^2} = \sqrt{13}$ $\qquad$ $F_1(\sqrt{13}+2, -3)$; $S_1(4|-3)$

$F_2(-\sqrt{13}+2, -3)$; $S_2(0|-3)$

$m = \pm\frac{b}{a} = \pm\frac{3}{2}$ $\qquad$ $a_1: y = \frac{3}{2}(x-2) - 3$

$a_2: y = -\frac{3}{2}(x-2) - 3$



c) $\sum_{i=0}^{n} 2^i = \frac{1-2^{n+1}}{1-2} = 2^{n+1} - 1$

Anfang: $n = 0$: $2^0 = 1 = 2^1 - 1$ ✓

Annahme: $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$

Schritt: $\sum_{i=0}^{n+1} 2^i = 2^{n+2} - 1$

L.S.: $\sum_{i=0}^{n} 2^i + 2^{n+1} = \underbrace{2^{n+1} - 1}_{\text{Annahme}} + 2^{n+1} = 2\cdot 2^{n+1} - 1 = 2^{n+2} - 1 \hat{=}$ Beh.