

1. a) $f(x) = x^3 - 3x^2 + 4$

Teiler: $\pm 1, \pm 2, \pm 4$

Long: $x = -1$

$(x^3 - 3x^2 + 4) : (x + 1) = x^2 - 4x + 4$

$x_{L/3} = 2$ dopp. \rightarrow Extrem

$f'(x) = 3x^2 - 6x$

$f'(x) = 0$

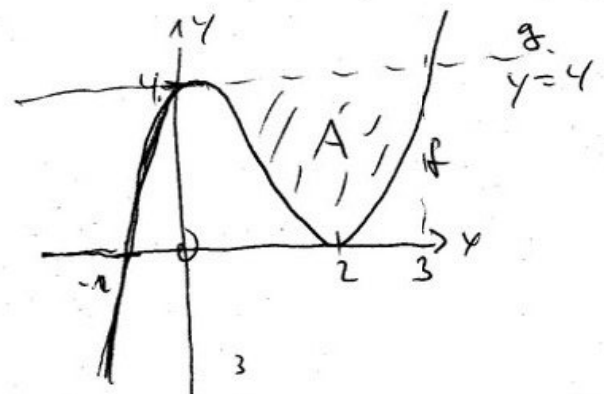
$x_1 = 0; x_2 = 2$

$y_1 = 4; y_2 = 0$

$f''(x) = 6x - 6$

$f''(0) = -6 < 0 \Rightarrow \text{Max}(0|4)$

$f''(2) = 6 > 0 \Rightarrow \text{Min}(2|0)$



$f = g$
 $x^3 - 3x^2 + 4 = 4$
 $x_1 = 0$
 $x_2 = 3$

$A = \int_0^3 (g - f) dx = \int_0^3 (-x^3 + 3x^2) dx = \left[-\frac{x^4}{4} + x^3 \right]_0^3 = \underline{\underline{\frac{27}{4}}}$

b) $f(x) = ax^3 - 3x^2 + 4$

$f'(x) = 3ax^2 - 6x$

$f''(x) = 6ax - 6$

$f''(x) = 0$

$x = \frac{1}{a}$

einfache NST
V&W
immer WP

$y = 0 = f(x)$
 $0 = 4 - \frac{2}{a^2}$
 $a = \pm \sqrt{2}$

2. a) $\cos \alpha = \frac{\overline{PQ} \cdot \overline{PR}}{PQ \cdot PR} = \frac{\begin{pmatrix} 8 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} -4 \\ 2 \end{pmatrix}}{10 \cdot \sqrt{20}} = \frac{-20}{10 \sqrt{20}} = -\frac{1}{\sqrt{10}}$
 $\alpha = 116,57^\circ$

b) $X(x|0) \quad \overrightarrow{XP} \cdot \overrightarrow{xQ} = 0$
 $\begin{pmatrix} 1-x \\ 1-0 \end{pmatrix} \cdot \begin{pmatrix} 9-x \\ 7-0 \end{pmatrix} = 0$
 $(1-x)(9-x) + 7 = 0$
 $x_1 = 2$
 $x_2 = 8$

c) $\overrightarrow{r_P} = \frac{1}{3}(\overrightarrow{r_Q} + \overrightarrow{r_A} + \overrightarrow{r_T})$
 $\overrightarrow{r_T} = 3\overrightarrow{r_P} - \overrightarrow{r_Q} - \overrightarrow{r_A} = \begin{pmatrix} -3 \\ -7 \end{pmatrix} \quad \underline{\underline{T(-3|-7)}}$

d) $M_{PA} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \quad \overrightarrow{n_{PA}} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$

$\overrightarrow{m_{PA}} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} + t \begin{pmatrix} -6 \\ 8 \end{pmatrix}$

$M_{PR} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} \quad \overrightarrow{n_{PR}} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\overrightarrow{m_{PR}} = \begin{pmatrix} -1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\overrightarrow{m_{PA}} = \overrightarrow{m_{PR}}$

$s = \frac{3}{2}$

$t = \frac{1}{2}$

S(2|8) : Mittelpunkt

3, a) $f(x) = x e^x$

$f'(x) = e^x + x e^x = (1+x) e^x$

$f''(x) = e^x + (1+x) e^x = (2+x) e^x$

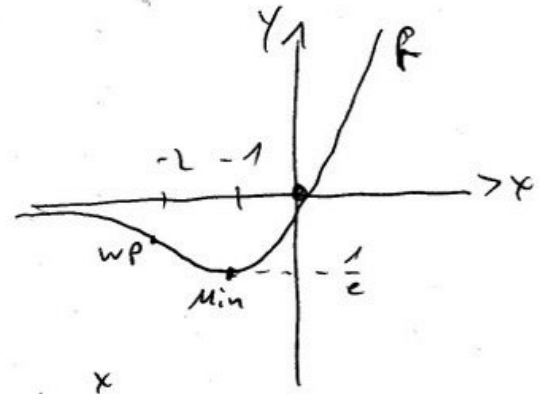
NST: $\underbrace{x}_{=0} \underbrace{e^x}_{\neq 0} = 0$
 $x=0$ einfach

Ext: $\underbrace{(1+x)}_{x=-1} e^x = 0$
 $y = -\frac{1}{e}$ $f''(-1) > 0 \Rightarrow \underline{\underline{\text{Min}(-1 | -\frac{1}{e})}}$

WP $(2+x) e^x$
 $x = -2$ einfach, VZW, WP $(-2 | -\frac{2}{e^2})$
 $y = -\frac{2}{e^2}$

$\lim_{x \rightarrow +\infty} x e^x = +\infty$
 $\downarrow \quad \downarrow$
 $+\infty \quad +\infty$

$\lim_{x \rightarrow -\infty} x e^x = 0$ (e-Fkt. stärker als Pol +)
 $\downarrow \quad \downarrow$
 $-\infty \quad 0$



b) $F(x) = (x+c) e^x$

$F'(x) = e^x + (x+c) e^x = \underbrace{(1+c+x)}_{f(x) = \frac{1}{e}} e^x$
 $\underline{\underline{c = -1}}$

$F(x) = (-1+x) e^x$

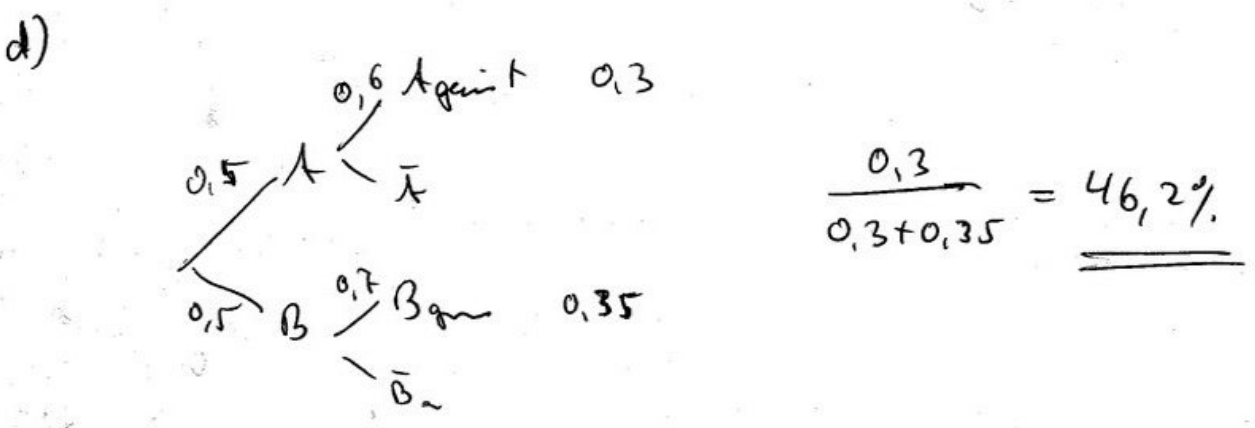
$A = \int_0^1 f(x) dx = [F(x)]_0^1 = (-1+1) e^1 - (-1+0) 1 = \underline{\underline{1}}$

4. $P_A = 0,6$; $P_B = 0,7$

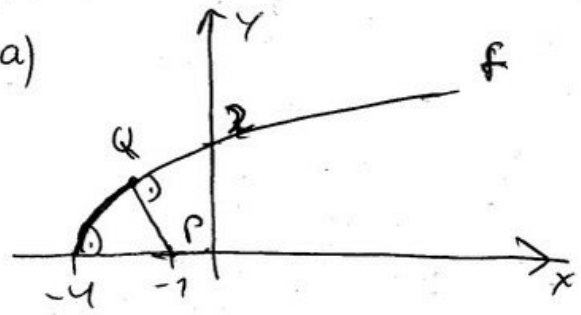
a) $P(\text{in 7 Versuche bis A genau 4 Mal}) = \binom{7}{4} p^4 q^3$
 $= \frac{7!}{4! 3!} 0,6^4 \cdot 0,4^3 = \underline{29,0\%}$

b) $P = P(2A, 0B) + P(1A, 1B) + P(0A, 2B)$
 $= 0,6^2 \cdot 0,3^2 + 2 \cdot 0,6 \cdot 0,4 \cdot 2 \cdot 0,7 \cdot 0,3 + 0,4^2 \cdot 0,7^2$
 $= \underline{\underline{31,2\%}}$

c) $P(n \text{ Versuche in A mind.}) \geq 0,9999$
 $1 - P(\text{Dmaldg}) \geq 0,9999$
 $1 - 0,4^n > 0,9999$
 $0,4^n < 0,0001$
 $n > 10,052$
Ab 11 Wiederholungen



5. a)



$$f'(x) = \frac{1}{2\sqrt{x+4}}$$

$$m_{PQ} = -\frac{1}{f'}$$

$$\frac{f(x)-0}{x+1} = -2\sqrt{x+4}$$

$$\sqrt{x+4} = -2\sqrt{x+4}(x+1)$$

$$\sqrt{x+4}(1+2(x+1)) = 0$$

$$x = -4$$

$$x = -\frac{3}{2}$$

$$y = \frac{1}{2}\sqrt{10}$$

$$Q(-\frac{3}{2} | \frac{1}{2}\sqrt{10})$$

b) I $\ln x = -0,5x$ | $f = g$

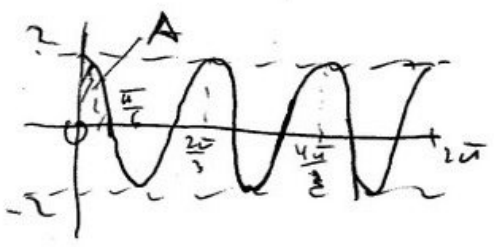
$\bar{u} \quad \frac{a}{x} = 2$ | $f' = -\frac{1}{g'}$

$\bar{u} \sim \bar{I}: \ln x = -1$

$$x = e^{-1}$$

$$\underline{a = 2x = 2e^{-1}}$$

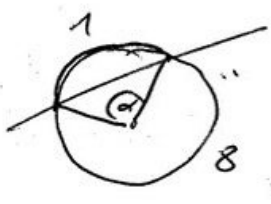
c)



$$A = \int_0^{\pi/6} 2 \sin(3x) dx = \left[-\frac{2}{3} \cos(3x) \right]_0^{\pi/6}$$

$$\underline{\underline{A = 2/3}}$$

d)



$$\alpha = \frac{1}{9} \cdot 360^\circ = 40^\circ$$

$$\underline{\underline{A_{seg} = A_s - A_d = \frac{\alpha}{360} \cdot r^2 \pi - \frac{1}{2} r^2 \sin \alpha}}$$

= 0,44

e)

$$\begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix} + t \begin{pmatrix} -3 \\ -2 \\ 2-3 \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \\ -3 \end{pmatrix} + s \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad \begin{cases} t = 2 \\ s = -4 \\ z = 4 \end{cases}$$

$$\underline{\underline{S(-4 | 5 | 5)}}$$

$$\cos \alpha = \frac{\begin{pmatrix} -3 \\ -2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}}{\sqrt{14} \cdot 3} \Rightarrow \underline{\underline{\alpha = 108^\circ}}$$