Mater # 13

1.a) $R = 5$, $M(-2|3)$, $P(1|y)$

$k: (x+2)^2 + (y-3)^2 = 25$

$x = 1:$ $\quad (y-3)^2 = 16$

$y - 3 = \pm 4$

$\underline{y = -1}$ $\quad (y<0)$

$P(1|-1)$

$\overrightarrow{MP} = \begin{pmatrix} 1+2 \\ -1-3 \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \end{pmatrix} \xrightarrow{\perp} \vec{v} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ $\qquad \underline{t: \vec{x} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} + s\begin{pmatrix} 4 \\ 3 \end{pmatrix}}$

$\underline{y = \frac{3}{4}(x-1) + 1}$

$\vec{r}_A = \vec{r}_M + 2\overrightarrow{MP} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 3 \\ -4 \end{pmatrix} = \begin{pmatrix} 4 \\ -5 \end{pmatrix}$

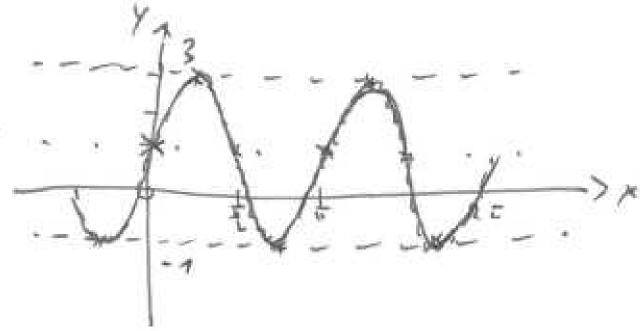
$\underline{\underline{A(4|-5)}}$

b) $y = A \cdot \sin(bx) + c$

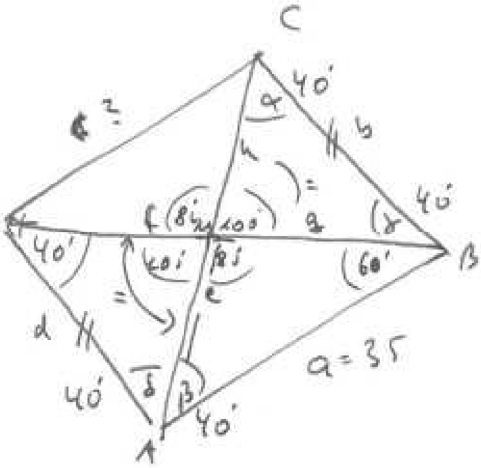
Periode π : $b = 2$

$y: \underbrace{-1 \,..\, 3}_{4} :$ $\rightarrow A = 2$ $\Big\} \; c = +1$

$\underline{\underline{y = 2\sin(2x) + 1}}$



c)



$e = AM$

$f = DM$

$\underline{e = f}$

$g = MB$

$h = MC$

$h = g$

$\frac{a}{\sin 80°} = \frac{e}{\sin 60°}$

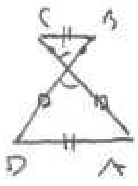
$e = \frac{\sin 60°}{\sin 80°} \cdot a = 30{,}778$

$\frac{g}{\sin 40°} = \frac{a}{\sin 80°}$

$g = \frac{\sin 40°}{\sin 80°} a = 22{,}8446 = h$

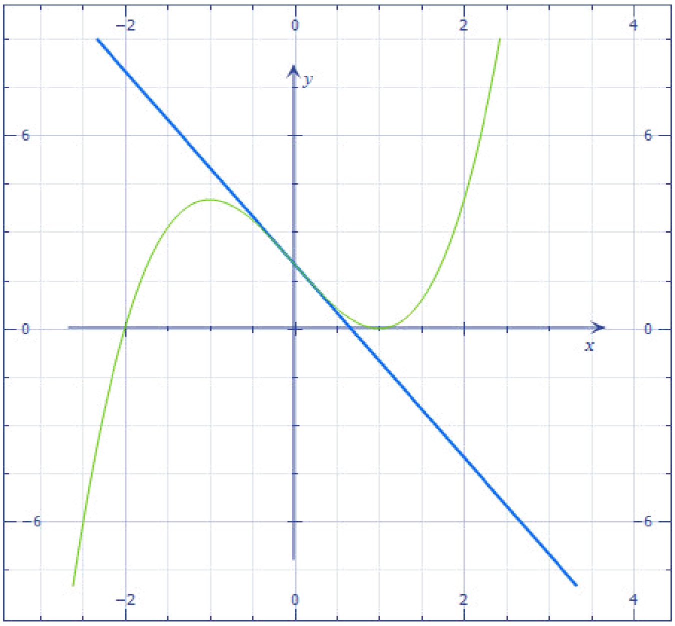
$c^2 = f^2 + h^2 - 2fh \cos 80°$

$\underline{\underline{c = 35}}$



$: AB = DC$

a) x=1 doppelte, x=-2 einfache Nullstelle:



$$f(x) = a(x-1)^2(x+2)$$

durch $P(2/4)$

$$4 = a(2-1)^2(2+2)$$

$$a = 1$$

$$f(x) = (x-1)^2(x+2) = x^3 - 3x + 2$$

b)

Die Funktion ist ein ungerades Polynom.

Somit sind D und W gleich R.

$$f'(x) = 3x^2 - 3$$

$$f''(x) = 6x$$

$$f' = 0$$

$$x = \pm 1 \qquad y_1 = 0 \quad y_2 = 4$$

$$f''(+1) > 0 \Rightarrow \text{Min}(1/0)$$

$$f''(-1) < 0 \Rightarrow \text{Max}(-1/4)$$

$$f'' = 0$$

$x = 0 \quad y = 2$ einfache Nullstelle,

also Vorzeichenwechsel, also WP(0/2)

c)

$$f'(0) = -3$$

$$t: y = -3x + 2$$

d)

$$A = \int_0^3 (f - t)\,dx = \int_0^3 \left(x^3 - 3x + 2 - (-3x + 2)\right)dx =$$

$$= \int_0^3 \left(x^3\right)dx = \left[\frac{1}{4}x^4\right]_0^3 = \frac{81}{4}$$

Mathe H13

3. 52 K, 2 – 10, B, D, K, A, H, K, P, K 4×13 K

2 – 9 : 2 – 9

10 – A : 10

a) Es sind noch 12 Herzkarten von 51 Karten da. $\frac{12}{51} = 23{,}5\,\%$

b) $\left(\frac{1}{52}\right)^2 = \frac{1}{2704} = 0{,}037\,\%$

c) 1)

$$P(\text{mind. einmal KK}) = 1 - P(\text{nie KK})$$
$$= 1 - \left(\frac{51}{52}\right)^{10} = 17{,}65\,\%$$

2)

$$P(\text{genau 2 mal HD}) = \binom{10}{2}\left(\frac{1}{52}\right)^2\left(\frac{51}{52}\right)^8$$
$$= \frac{10!}{2!\,8!}\left(\frac{1}{52}\right)^2\left(\frac{51}{52}\right)^8$$
$$= 1{,}425\,\%$$

d)

$$P(\text{mind. einmal KB}) > 8\,\%$$
$$1 - P(\text{nie KB}) > 0{,}8$$
$$\left(\frac{51}{52}\right)^n < 0{,}2$$
$$n > \log_{\frac{51}{52}} 0{,}2 = 82{,}88$$

mindestens 83 mal.

e) $8 = \underline{2+6} = 3+5 = 4+4 = 5+3 = 6+2$: 5 Möglichkeiten

eine günstige

$P = \frac{1}{5} = 20\,\%$

Mathe H13

4. $f(x) = (1-x)^2 e^{-x}$

a) $\lim\limits_{x \to +\infty} \underset{\infty \quad \cdot \quad 0}{(1-x)^2 e^{-x}} = \underline{0}$ $\quad e^x$ stärker als jede Potenz

pos. x-Achse ist waag. A.

$\lim\limits_{x \to -\infty} \underset{\infty \quad \infty}{(1-x)^2 e^{-x}} = \infty$

$f'(x) = 2(1-x)(-1)e^{-x} + (1-x)^2 e^{-x}(-1)$

$\underline{f'(x) = -(x-3)(x-1)e^{-x}}$

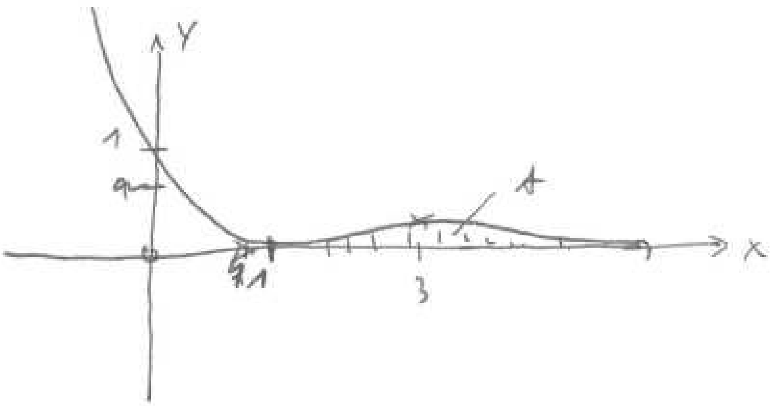
$\underline{f'(x) \text{ Nullstellen bei } 1 \text{ und } 3}$ $\quad$ VzT.

x	0	1	2	3	4
$f'(x)$	$-$	0	$+$	0	$-$

$\searrow \rightarrow \nearrow \quad \rightarrow \searrow$

$\underline{\text{Min}(1|0)}$

$\underline{\text{Max}(3|4e^{-3})}$



b) $F(x) = -(x^2+1)e^{-x}$

$\underline{F'(x)} = -2x e^{-x} - (x^2+1)e^{-x}(-1)$

$= (x^2 - 2x + 1)e^{-x}$

$= (x-1)^2 e^{-x} = \underline{f(x)}$

c) $\underline{\underline{A}} = \int_1^\infty f(x)\,dx = \lim\limits_{a \to \infty} F(a) - F(1)$

$= \lim\limits_{a \to \infty} \underbrace{-\underset{\infty}{(a^2+1)}\underset{0}{e^{-a}}}_{0 \text{ s.o.}} + \frac{2}{e} = \underline{\underline{\frac{2}{e}}}$

Mathe HB

5. $A(3|0|3)$ $B(4|-2|1)$ $C(-1|-6|4)$

a) $\overline{AB} \times \overline{AC} = \begin{pmatrix}1\\-2\\-2\end{pmatrix} \times \begin{pmatrix}-4\\-6\\1\end{pmatrix} = \begin{pmatrix}-14\\7\\-14\end{pmatrix} \sim \begin{pmatrix}2\\-1\\2\end{pmatrix}$

$E: \quad 2x + y + 2z + d = 0$

$A: \quad 6 - 0 + 6 + d = 0$

$\underline{E: \quad 2x - y + 2z - 12 = 0}$

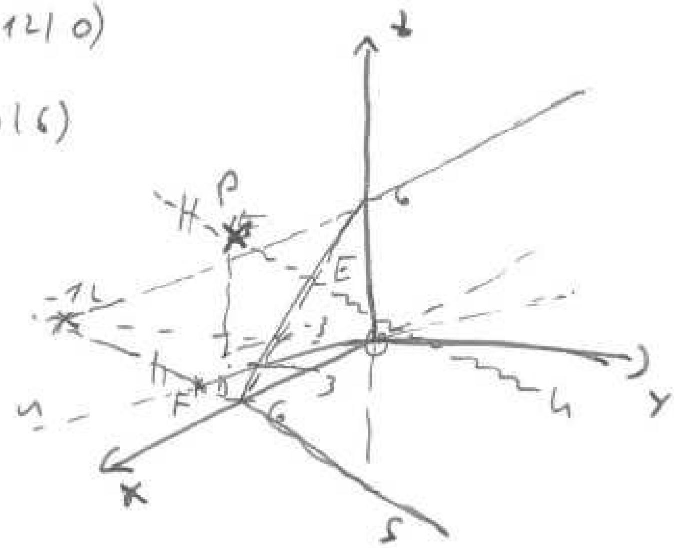
$E: \quad \vec{x} = \begin{pmatrix}3\\0\\3\end{pmatrix} + s\begin{pmatrix}1\\-2\\-2\end{pmatrix} + t\begin{pmatrix}-4\\-6\\1\end{pmatrix}$

b) $X\ (y = z = 0): \quad 2x - 12 = 0 \qquad X(6|0|0)$

$\underline{x = 6}$

$Y(x = z = 0): \qquad Y(0|-12|0)$

$Z\ (x = y = 0): \qquad Z\ (0|0|6)$



c) $y = 2x - 12$

$\left.\vec{x} = \begin{pmatrix}6\\0\\0\end{pmatrix} + t\begin{pmatrix}6\\12\\0\end{pmatrix}\right\} s$

d) $P(3|-3|6)$

$h: \quad \vec{x} = \begin{pmatrix}3\\-3\\6\end{pmatrix} + t\begin{pmatrix}6\\12\\0\end{pmatrix}$

e) $n \perp s$ durch $O: \quad y = -\frac{1}{2}x$

$n \cap s: \qquad -\frac{1}{2}x = 2x - 12$

$x = \frac{24}{5} \qquad y = -\frac{12}{5} \qquad F\left(\frac{24}{5} \middle| -\frac{12}{5}\right)$

$\underline{d} = OF = \frac{1}{5}\sqrt{24^2 + 12^2} = \underline{\frac{1}{5}\sqrt{577}} = 4{,}804$